TILTING URANUS

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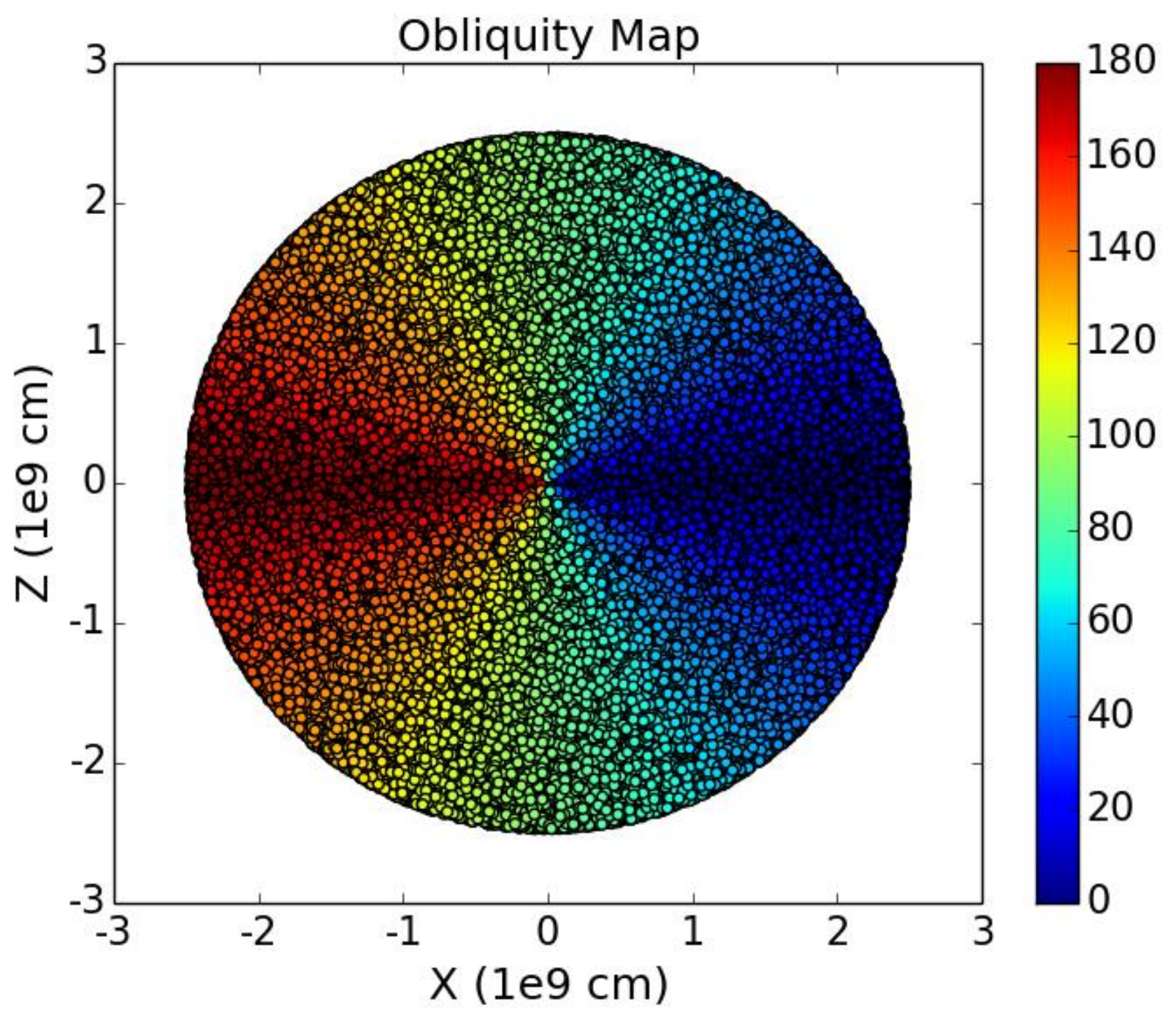
WITH A COLLISION

Background

Uranus' obliquity, the angle between its spin axis and the normal to its orbital plane, is about 98°. The most accepted model to tilting Uranus over is a polar strike of an Earth sized object[1] or two or more smaller collisions [2]. These models assume Uranus' initial obliquity was 0°. Since Uranus is made up of mostly ice and rock (90%–70%[3,4]), it must have been built up by a series of collisions[5,6,7]. These collisions impart angular momentum onto Uranus which would determine its final spin state**. How likely was Uranus to have an obliquity near zero degrees prior to a giant impact?**



Obliquity

The angular momentum imparted onto a planet is:

$$L_i = m_i(\mathbf{r} \times \mathbf{v})$$

m_i is the mass of the impactor, $\boldsymbol{r}$ is the collision position, and $\boldsymbol{v}$ is the impactor's velocity. The obliquity is given by: $cos(\varepsilon) = L_z/|L|$. We test two possible distribution of impactors: isotropic and planar (Figs. 2 & 3). A planar distribution is where the impactors are located within the disk of thickness about the diameter of the planet. **Both the isotropic and planar distributions show that untilted planets are rare.** Both distributions are comparable for tilting a planet around 90°.

Fig. 1 (left): A map of Uranus' cross section where each dot signifies its final obliquity, in degrees, after a planar strike from one 1 $M_\oplus$ impactor. $\varepsilon_0 = 0$.

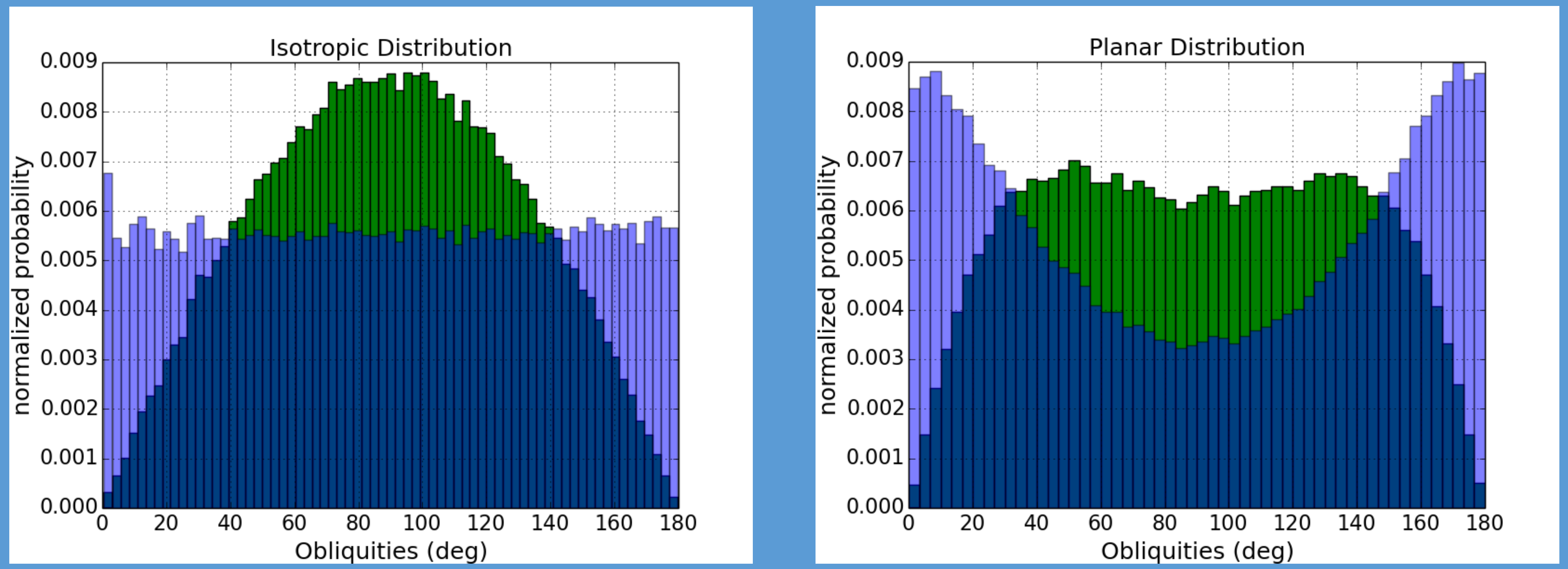
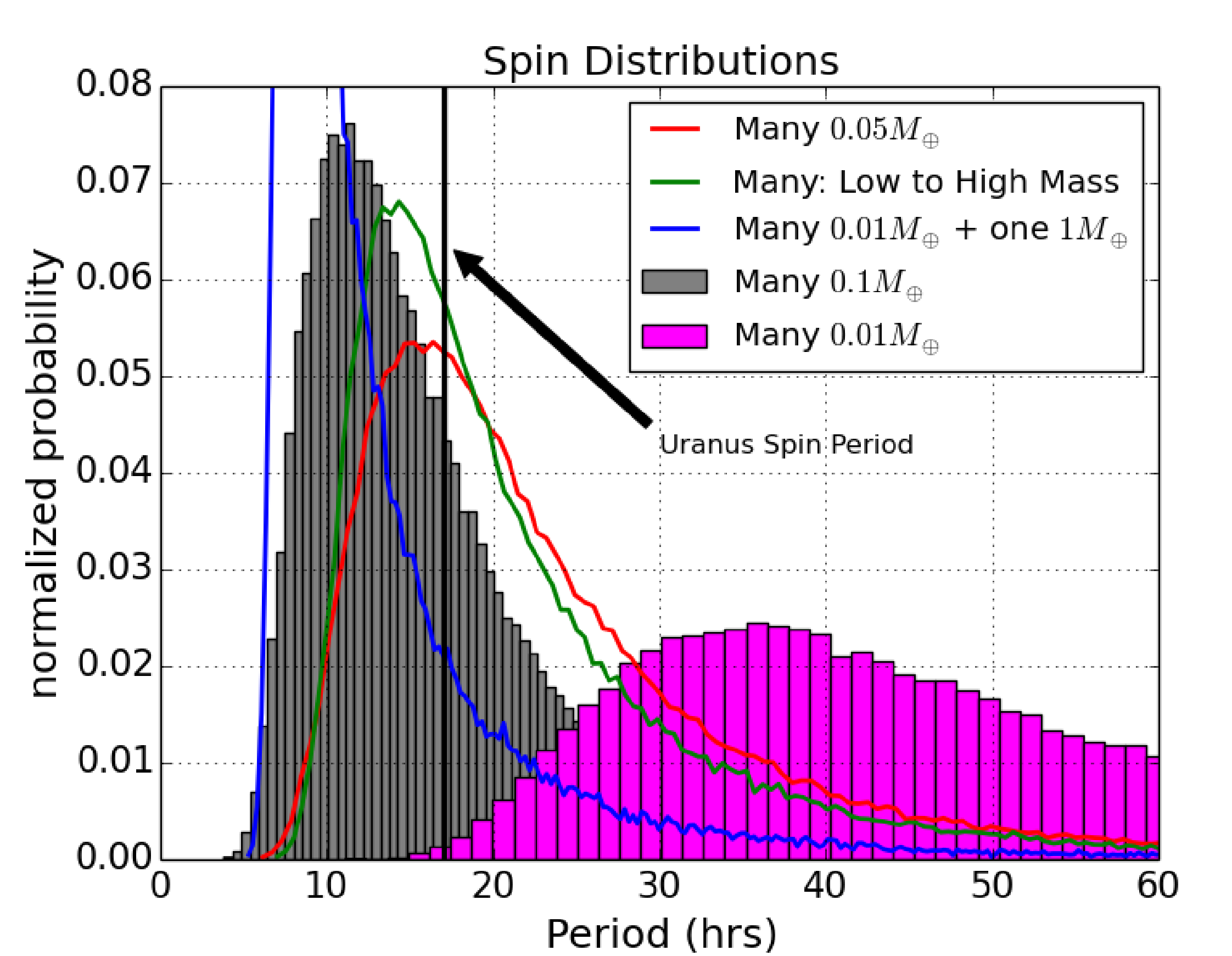


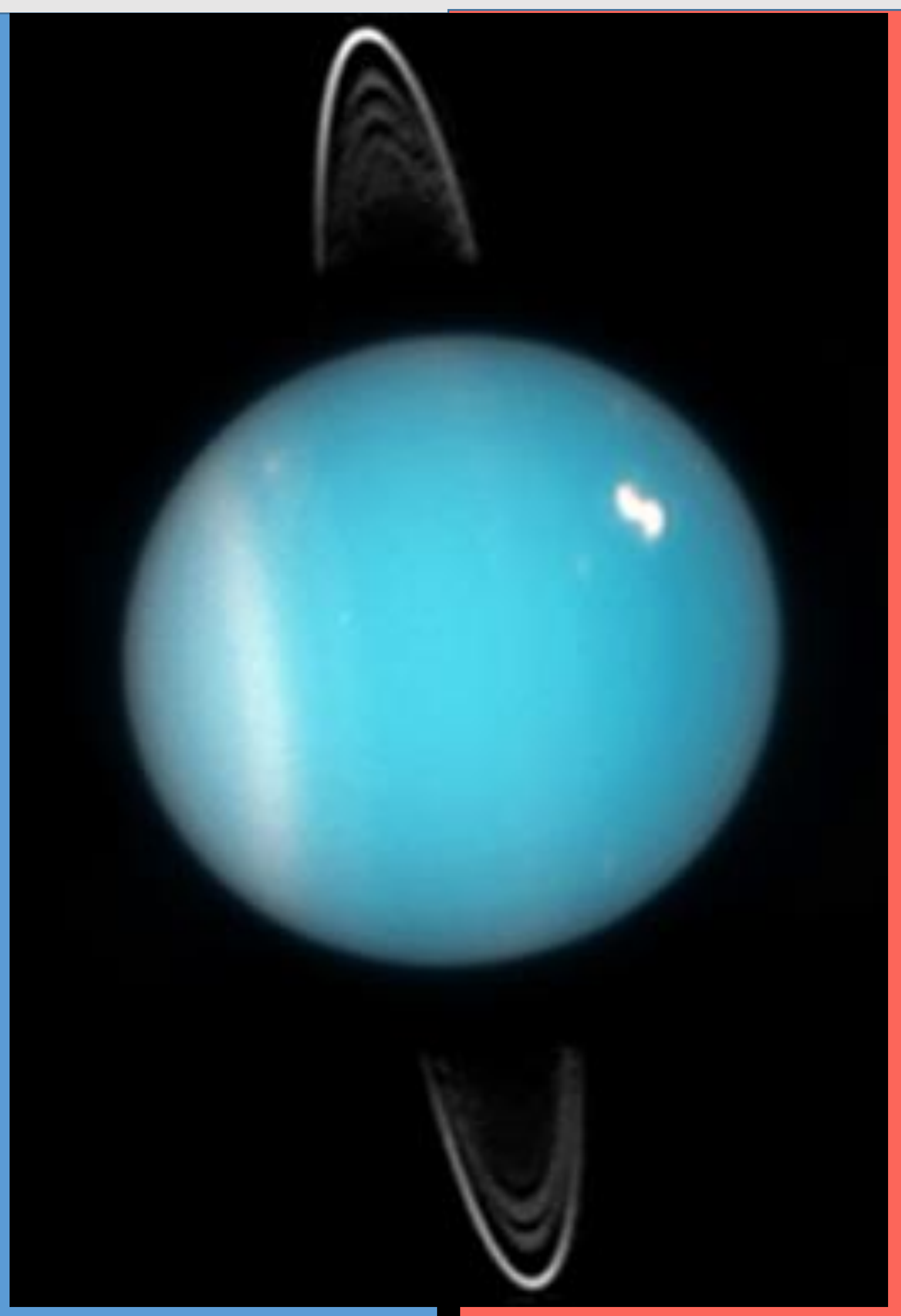
Fig. 2 (left) shows the distribution of obliquities (green) for a planet built up by collisions of an isotropic distribution of impactors. Fig. 3 (right) has the impactors striking the planet parallel to its orbital plane. The impactors fall onto the planet at the planet's escape velocity, and the planet's initial mass and spin are both negligible. The blue transparent overlays are the probabilities weighted by sin(ε)--the obliquity densities. Deviations from a uniform distribution shows bias, and for the planar distribution there is a bias towards obliquities near 0° or 180°.



Spin

What does the impactor mass distribution tell us about the spin period distribution of the planet? Figure 4 shows the spin distributions for Uranus being built up by a number of mass distributions. The width of the distributions depend on the root mean square of the angular momentum imparted by the impactors. Fine tuning is required to generate a spin distribution that peaks at Uranus' current rotation period. The spin distribution generated from planar impactors resulted in better statistics than impactors from an isotropic distribution. We will refine these distributions by adding in gas accretion for both Uranus and Neptune, and exploring alternative mass distributions consistent with models of planetary disks.

Fig. 4 (left): The sum of the masses of the impactors is Uranus' current mass 14.5 $M_\oplus$. The black vertical line is Uranus' current spin period of 17 hours. Red and green curves provide good fits.



WITHOUT A COLLISION

Background

Jupiter and Saturn are gas giants composed of mostly hydrogen and helium. As the gas from the accretion disk collapsed onto the forming planet, angular momentum was conserved, and so Jupiter's and Saturn's obliquities would be driven towards zero. Jupiter's obliquity is low (3°) while Saturn's is not (27°). **Saturn's tilt can best be explained by a secular resonance between the precession frequencies of Saturn's spin axis and Neptune's orbital pole[8,9]. Can we tilt Uranus over similarly?**

Model

The precessional motion of a planet is described accordingly[10]:

$$\frac{d\hat{\sigma}}{dt} = \alpha(\hat{\sigma} \times \hat{n})(\hat{\sigma} \cdot \hat{n})$$

$\hat{\sigma}$ points in the direction of the total angular momentum of the Uranian system, $\hat{n}$ points in the direction of Uranus' orbital angular momentum, and α is the precession frequency near zero obliquity.

$$\alpha \propto \frac{\Omega^2 J_2}{\omega}$$

$\Omega \propto r^{-\frac{3}{2}}$ is the orbital angular speed, J_2 is the quadrupole gravitational moment, and ω is the spin angular speed. The precession period is given by:

$$T = \frac{2\pi}{\alpha \cos(\varepsilon)}$$

Uranus' current precession period is about 210 Myr. For low tilts ($\cos(\varepsilon) \approx 1$) and including the torque of the Uranian moons, Uranus' precession period is about 29 Myr. There are no orbital precession periods today that match Uranus' spin precession period at any obliquity (Table 1). **Could Uranus' precession rate have been more rapid in the past?**

Uranus and Neptune probably formed closer to the Sun as the protoplanetary disk was denser, and then later migrated outward[12]. Thommes et al. (1999, 2002, 2003)[13,14,15] argue that Uranus and Neptune probably formed in between Jupiter and Saturn (4-10 au) as the timescale for formation beyond Saturn would take too long. Tilting Uranus while interior to Saturn is appealing, as Uranus' axial precession frequency would have been much smaller. **With Uranus at 7 au its axial precession period would be 1.45 Myr. This could result in a resonance with Neptune if Neptune was ejected beyond Saturn first.**

Planets	Inclination Period (Myr)
Jupiter	∞
Saturn	0.05
Uranus	0.45
Neptune	1.91

Table 1: The four fundamental inclination frequencies measured today[11].

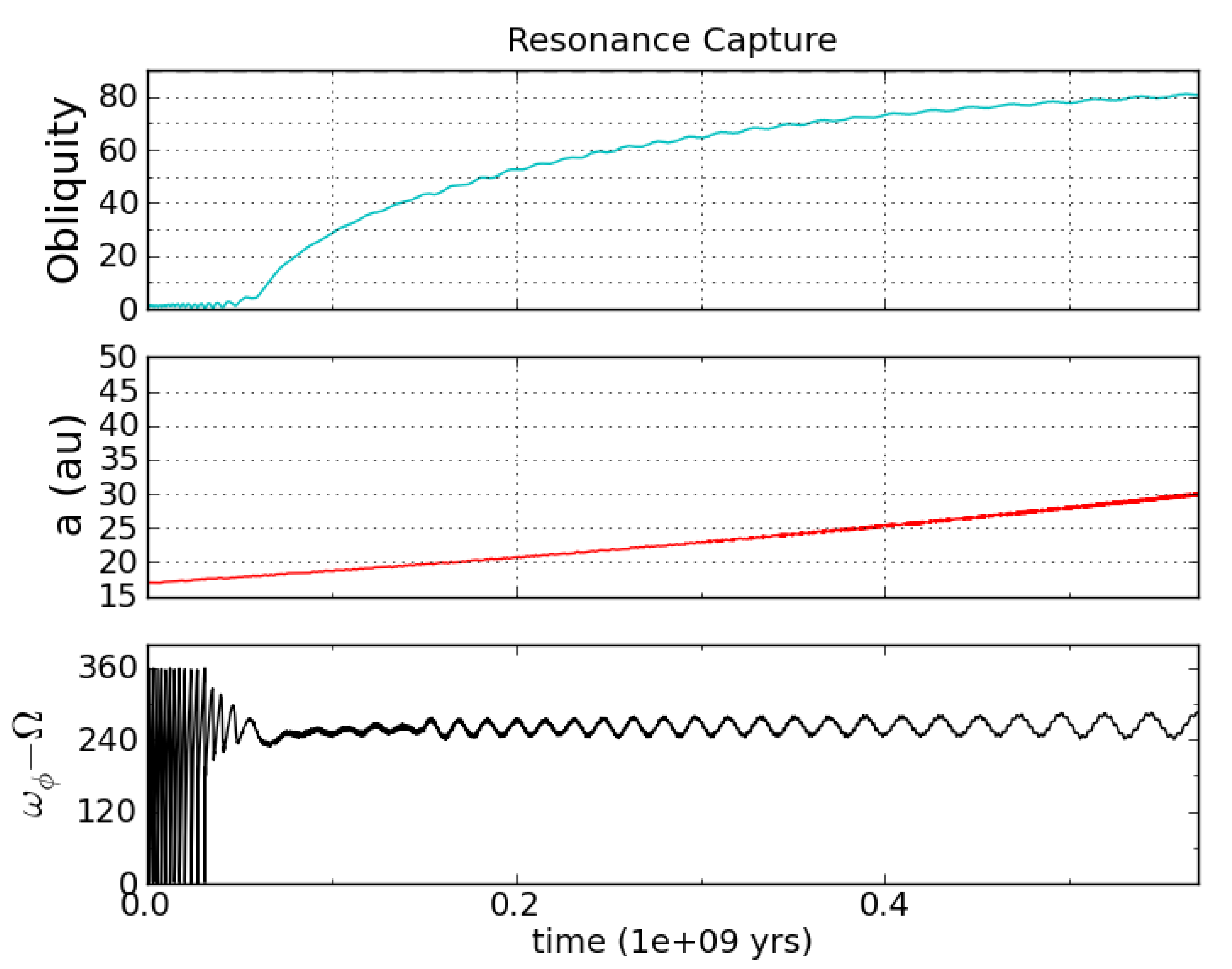


Fig. 5: Jupiter (4.3 au), Uranus (6.3 au), Saturn (8.3 au). Uranus' radius is 50% larger assuming the tilting occurred 4 billion year ago[16]. Top panel describes Uranus' obliquity. Middle panel shows Neptune's migration. Bottom panel shows the difference between Uranus' azimuthal angle and Neptune's longitude of ascending node, which depicts the resonance.

Non-collisional Tilting

As Neptune migrates outward, its orbital precession period slows resulting in a resonance capture with Uranus (Fig. 5). As Uranus tilts its axial precession period slows resulting in the two bodies remaining in resonance during Neptune's migration. The process of tilting Uranus to a high obliquity takes a few 100 Myr. This timescale may be too long for Uranus to remain between Jupiter and Saturn, and we are investigating how to reduce it. We also find that resonance capture is rare if Uranus' initial obliquity is greater than about 10°. We will refine this estimate by quantifying capture statistics, and running accretion simulations to test the likelihood of a low early obliquity. Although trapping is unlikely, a resonant kick can tilt Uranus by $\sim 45^\circ$. We will explore planetary migration scenarios in which a series of kicks could tilt Uranus significantly. We also expect dramatic differences to prograde and retrograde spins.

REFERENCES AND ACKNOWLEDGEMENT

1. Parisi, M. G. and Brunini, A. , 1997, Planet. Space Sci., 45:181 **2.** Morbidelli, A., et al., 2012, Icarus, 219:737 **3.** Podolak, M., et al., 1995, Planet. Space Sci., 43:1517 **4.** Podolak, M., et al., 2000, Planet. Space Sci., 8:143 **5.** Dones, L. and Tremaine, S., 1993, Science, 259:350 **6.** Lissauer, J. J. and Safronov, V. S., 1991, Icarus, 93:288 **7.** Lissauer, J. J. and Kary, D. M., 1991, Icarus, 94:126 **8.** Ward, W. R. and Hamilton, D. P., 2004, AJ, 128:2501 **9.** Hamilton, D. P. and Ward, W. R., 2004, AJ, 128:2510 **10.** Tremaine, S., 1991, Icarus, 89:95 **11.** Murray, C. D. and Dermott, S. F., 1999, Solar System Dynamics **12.** Fernandez, J. A. and Ip, W.-H., 1984, Icarus, 58:109 **13.** Thommes, E. W., et al., 1999, Nature, 402:635 **14.** Thommes, E. W., et al., 2002, AJ, 123:2862 **15.** Thommes, E. W., et al., 2003, Icarus, 161:431 **16.** Bodenheimer, P. and Pollack, J. B., 1986, Icarus, 67:391

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